

## UNIT CONTENTS – SENIOR ASTROPHYSICS

### UNIT 10 - FUNDAMENTAL KINEMATICS II & INTRO TO CALCULUS (DIFFERENTIAL & INTEGRAL) 5 HOURS (10 WEEKS)

#### UNIT CONTENT

Conservation of momentum in 1 and 2 dimensions (vector analysis); Definition of angular velocity  $\omega = \frac{d\theta}{dt}$ ,  $\omega = \frac{2\pi}{T}$  and relationship to linear velocity  $v = \omega r$ ; Definition of angular acceleration  $a_r = \frac{v^2}{r} = \omega^2 r$ ; Recap Newton's universal law of gravitation and escape and critical velocities; Recap centripetal force; Newton's 1<sup>st</sup> law extended to angular momentum & external torque; recap rotational mechanics; Review of gradients, average and instantaneous rates of change  $\frac{\Delta y}{\Delta x}$ ; Concept of a limit  $\lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x}$ ; Derivative by first principles (limit theory); Application of the derivative principles e.g.  $\frac{ds}{dt}$  &  $\frac{dv}{dt}$ ; Rules for differentiation (polynomial & exponential functions) – power functions (product & quotient rules), higher-order derivatives (chain rule); Curvilinear motion (parametric equations); Area under a curve (e.g. velocity-time graph); Relationship between the formula for the area under the curve of a function and the function (i.e.  $A = \frac{1}{2}x^2$  and  $f(x) = x$ ); General pattern for integration  $\frac{x^{n+1}}{n+1}$ ; Integrand function as  $F(x) = \int f(x)dx$ ; Application of integration principles e.g.  $s = \int v dt$  &  $v = \int a dt$ ; Concept of a definite integral  $\int_a^b f(x)dx = F(b) - F(a)$ ; Higher-order integrals – substitution methods; Integration by parts; Magnitudes of resolved vectors for displacement, velocity & acceleration; Direction vectors for displacement, velocity and acceleration; Related rates w.r.t. time e.g.  $\frac{dF}{dt}$ ,  $\frac{dr}{dt}$ ; Work done by variable forces  $W = \int_a^b f(x) dx$ ; Derivation of gravitational potential energy between two masses (using calculus).

#### ASSESSMENT

Supervised Written (Examination) – Unit 10 Parts A & B. Duration 2 hrs. 20 mins total.

## UNIT 11 – THE NATURE OF STARS & DISTANCE RELATIONS

21 hours (6 weeks)

### UNIT CONTENT

Measuring the distances of stars using stellar parallax (terrestrial & space); Intrinsic brightness (luminosity -  $L$ ) and apparent brightness ( $b$ ) (flux -  $F$ ); Brightness factor  $100^{\frac{1}{5}} \approx 2.512$ ; Using photometry to measure a star's apparent brightness (flux); Relationship between a star's apparent brightness, its luminosity and distance  $b = \frac{L}{4\pi d^2}$ ; Calculating a star's luminosity from its apparent brightness  $\frac{L}{L_{\odot}} = \left(\frac{d}{d_{\odot}}\right)^2 \frac{b}{b_{\odot}}$ ; Magnitude scales - Apparent magnitude ( $m$ ) and Absolute magnitude ( $M$ ); Magnitude difference related to brightness ratio  $m_2 - m_1 = 2.5 \log\left(\frac{b_1}{b_2}\right)$ ; Relationship between a star's apparent magnitude and absolute magnitude  $m - M = 5 \log d - 5$ ; Relation between a star's colour and temperature (blackbody curves and dominant wavelength); UBV photometry; Absorption line spectra & spectral classes & types – OBAFGKM (spectral) sequence; Strength of absorption lines; Stellar radii, luminosity & surface temperature  $L = 4\pi R^2 \sigma T^4$ ,  $\frac{L}{L_{\odot}} = \left(\frac{R}{R_{\odot}}\right)^2 \left(\frac{T}{T_{\odot}}\right)^4$ ; Hertzsprung-Russell (H-R) diagrams; Star varieties - main Sequence (V), sub-giants (IV), giants (III), bright giants, (II), supergiants (Ia & Ib), white & brown dwarfs; Determining a star's size from its spectrum; Luminosity classes; Spectroscopic parallax; Binary star systems - Kepler's 3<sup>rd</sup> law for binary star systems  $M_1 + M_2 = \frac{a^3}{T^2}$ ; Radial velocity curves for binary systems and light curves of eclipsing binaries; Main-sequence masses & mass-luminosity relation; Total flux of a star  $F_{\text{bol}} = \int_0^{\infty} F_{\lambda} d\lambda$ ; Apparent bolometric magnitude  $m_{\text{bol}} = C_{\text{bol}} - 2.5 \log F_{\text{bol}}$ ; Relation between absolute bolometric magnitude and luminosity  $\frac{L}{L_{\odot}} = 10^{0.4(4.74 - M_{\text{bol}})}$ ; Filter systems (e.g. *Johnson-Cousins system*) and Multicolour photometry; Colour indices (e.g.  $B - V = m_B - m_V$ ); Relation between the surface temperature  $T$  and the colour index  $T \approx \frac{9000 \text{ K}}{(B-V)+0.93}$ ; Bolometric corrections (e.g.  $BC = m_{\text{bol}} - m_V = M_{\text{bol}} - M_V$ ); Atmospheric extinction (dimming)  $m_V(\text{above atmosphere}) \approx m_V - 0.2 \text{ sec } Z$ .

### ASSESSMENT

CLEA computer simulation activity "Photoelectric Photometry" and exercise questions.

### ADDITIONAL ASSESSMENT OPPORTUNITIES

- ☼ GHOU "Brightness Vs. Distance" unit
- ☼ GHOU "Spherical Distribution" unit
- ☼ GHOU "Distance, Luminosity Vs. Apparent Brightness" unit

## UNIT 12 - BIRTH & LIFE OF STARS

21 hours (6 weeks)

### UNIT CONTENT

Nebulae & interstellar medium; Emission nebulae – excitation of hydrogen (H II regions); Dark & reflection nebulae (including Bok globules); Interstellar extinction & reddening; Evolution of protostars and pre-main-sequence stars; High & low mass; T Tauri stars (bipolar outflow & Herbig-Haro objects); Accretion disks; Star clusters; Cluster plots on HR diagrams to determine the age of the cluster; Molecular clouds; Supernova remnants and nebulae; Main-sequenced stars (ZAMS) – zero-age main sequence; Evolution of stars between  $0.08$  and  $0.4 M_{\odot}$  and  $M > 0.4 M_{\odot}$ ; Main-sequence lifetimes; Post-main-sequence evolution; Core helium fusion, helium flash, electron degeneracy & position on H-R diagram; Star clusters – ages & turnoff points; Globular & open clusters; Population I & II stars; Variable stars – Cepheid variability; Cepheids as a measure of distance using  $M = m - 5 \log(d/10)$ ; Period-Luminosity relation; Binary stars & mass transfer.

### MATHEMATICAL BASE

**Star Formation:** Density of molecular hydrogen ( $H_2$ ) clouds  $\rho_{mc} \approx 2m_p n_{mc}$ ; Ratio of final and initial radius of a molecular cloud (one solar mass)

$$\frac{R_{\odot}}{R_{mc}} = \left(\frac{\rho_{mc}}{\rho_{\odot}}\right)^{\frac{1}{3}}; \text{ Free-fall time } t_{\text{ff}} = \left(\frac{3\pi}{32G\rho_0}\right)^{\frac{1}{2}} \text{ and pressure gradient time } t_{\text{press}} = \frac{r_0}{c_s} \text{ where } c_s = \left(\frac{\gamma kT}{\mu m_p}\right)^{\frac{1}{2}}; \text{ Jeans length } r_j = \left(\frac{3\pi\gamma kT}{32G\rho_0\mu m_p}\right)^{\frac{1}{2}} \text{ or}$$

$$r_j \approx 2000 \text{ AU} \left(\frac{T}{10 \text{ K}}\right)^{\frac{1}{2}} \left(\frac{\rho_0}{3 \times 10^{-15} \text{ kg m}^{-3}}\right)^{-\frac{1}{2}}; \text{ Jeans mass } M_j \approx 0.2 M_{\odot} \left(\frac{T}{10 \text{ K}}\right)^{\frac{3}{2}} \left(\frac{\rho_0}{3 \times 10^{-15} \text{ kg m}^{-3}}\right)^{-\frac{1}{2}}; \text{ Rotation speeds } v_f = \left(\frac{r_0}{r_f}\right) v_0; \text{ Radius of disc after collapse } r_f = \frac{v_0^2 r_0^2}{GM}.$$

## UNIT 12 - BIRTH & LIFE OF STARS (CONT.)

### MATHEMATICAL BASE

**Stellar Interiors - Equations of Stellar Structure:** Derivation of the equation of hydrostatic equilibrium  $\frac{dP}{dr} = -\frac{GM_r \rho_r}{r^2}$ ; Deriving an equation to estimate the central pressure of a star  $P_c \approx 2 \frac{GM_\odot \rho_\odot}{R_\odot} \approx \frac{8\pi}{3} G \rho_\odot^2 R_\odot^2$ ; Equation of mass continuity  $\frac{dM}{dr} = 4\pi r^2 \rho(r)$ ; Relating pressure to density using the equation of state  $P(r) = \frac{\rho(r)kT(r)}{\mu m_p}$ ; Radiation pressure exerted by photons  $P_{\text{rad}}(r) = \frac{a}{3} [T(r)]^4$ ; Estimation of the central temperature of a star using the ideal gas law as the equation of state  $T_c \approx P_c \frac{\mu_\odot m_p}{\rho_\odot k} \approx \frac{2GM_\odot \mu_\odot m_p}{R_\odot k}$ ; Central temperature for sphere of ideal gas in hydrostatic equilibrium  $T_c \propto \frac{M\mu}{R}$ .

**Energy Generation in Stars:** The equation of energy generation  $\frac{dL}{dr} = 4\pi r^2 \rho(r) \epsilon(r)$ ; Derive the gravitational potential energy of a mass shell  $dU = -\frac{GM_r}{r} dm$  and  $U = -\frac{3}{5} \frac{GM^2}{R}$ ; Gravitational potential energy of the Sun as  $U_\odot = -q \frac{GM_\odot^2}{R_\odot}$ ; Kelvin-Helmholtz time  $t_{KH} \equiv \frac{|U_\odot|}{L_\odot}$ ; Mass deficit and Einstein's mass energy equation  $\Delta E = (\Delta m)c^2$ ; Energy release from fusion  $E_{\text{fus}} = \frac{N_H}{4} \Delta E$  and fusion timescale  $t_{\text{fus}} = \frac{E_{\text{fus}}}{L_\odot}$ ; Potential energy of two protons separated by a distance  $r$   $U = \frac{e^2}{4\pi\epsilon_0 r}$ ; Typical kinetic energy of a proton at the centre of the  $\langle E \rangle = \frac{3kT_c}{2}$ ; de Broglie wavelength for a proton  $\lambda_{dB} = h(2m_p E)^{-\frac{1}{2}}$ ; Minimum kinetic energy at which tunnelling has a significant probability  $E \approx \left(\frac{e^2}{4\pi\epsilon_0}\right)^2 \frac{2m_p}{h^2}$ ; The net result of the PP chain being  $4p \rightarrow {}^4_2\text{He} + 2e^+ + 2\nu_e + 2\gamma$ ; The net result of the CNO cycle being  $4p \rightarrow {}^4_2\text{He} + 2e^+ + 2\nu_e + 3\gamma$ ; Triple alpha process  ${}^4_2\text{He} + {}^4_2\text{He} \rightarrow {}^8_4\text{Be} + \gamma$  and  ${}^4_2\text{He} + {}^8_4\text{Be} \rightarrow {}^{12}_6\text{C} + \gamma$ .

### ADDITIONAL ASSESSMENT OPPORTUNITIES

- ☼ GHOU "Cepheid Variable Stars" unit
- ☼ GHOU "The Science of Cepheids" unit

## UNIT 13 – STELLAR REMNANTS

21 hours (6 weeks)

### UNIT CONTENT

Old low-mass stars becoming Supergiants ( $0.4 - 4 M_{\odot}$ ); Horizontal branch stages of red giant stars; Asymptotic giant branch (AGB) stars – structure; Planetary nebulae; White dwarfs; Properties; Mass-radius relation; Intermediate-Mass and High Mass stars; Heavy element fusion reactions; Supernova explosions; Supernovae types (Ia, Ib, Ic, II); Decay of supernova; Remnants of supernovae - neutron stars and pulsar.

### MATHEMATICAL BASE

Average density of a white dwarf as a ratio to solar density  $\rho_{\text{wd}} = \frac{M_{\text{wd}}}{M_{\odot}} \left( \frac{R_{\odot}}{R_{\text{wd}}} \right)^3 \rho_{\odot}$ ; Derivation of central pressure as a ratio to central solar pressure  $P_c \sim \left( \frac{M_{\text{wd}}}{M_{\odot}} \right)^2 \left( \frac{R_{\odot}}{R_{\text{wd}}} \right)^4 P_{c,\odot}$ ; Electron degeneracy pressure; Quantum states of an electron in a hydrogen atom (principle  $n$ ), (orbital  $m_l$ ), (orbital magnetic  $l$ ) and (orbital magnetic  $m_s$ ); Pauli exclusion principle; Heisenberg uncertainty principle  $\Delta x \Delta p \geq \hbar$ ; number density (of degenerate electrons)  $n_e$ ; Uncertainty of a degenerate electron's location as  $\Delta x \sim V^{1/3} \sim n_e^{-1/3}$ ; Uncertainty of a degenerate electron's momentum as  $\Delta p \sim \frac{\hbar}{\Delta x} \sim \hbar n_e^{1/3}$ ; Velocity (Heisenberg speeds) of nonrelativistic degenerate electrons as  $\Delta v = \frac{\Delta p}{m_e} \sim \frac{\hbar n_e^{1/3}}{m_e}$ ; Velocity of electrons due to thermal motion  $v_{\text{th}} \sim \left( \frac{kT}{m_e} \right)^{1/2}$ ; Pressure contributed by thermal pressure as  $P_{\text{th}} = n_e kT \sim n_e m_e v_{\text{th}}^2$ ; Pressure contributed by degenerate electrons  $P_{\text{degen}} \sim n_e m_e (\Delta v)^2 \sim n_e m_e \left( \frac{\hbar n_e^{1/3}}{m_e} \right)^2 \sim \hbar^2 \frac{n_e^{5/3}}{m_e}$ ; Mass-radius relationship -  $R \sim \frac{\hbar^2}{G m_e m_p^2} \left( \frac{M}{m_p} \right)^{-1/3}$ ; Increasing average density rapidly with mass -  $\langle \rho \rangle \sim \frac{M}{R^3} \sim \frac{G^3 m_e^3 m_p^5}{\hbar^6} M^2$ ; Number & critical mass density of relativistic degenerate electrons  $n_e \sim \left( \frac{cm_e}{\hbar} \right)^3$  and  $\rho \sim \frac{2c^3 m_e^3 m_p}{\hbar^3}$ ; Mass when degenerate electrons becomes relativistic  $M \sim \left( \frac{\hbar^3 c^3}{G^3 m_p^4} \right)^{1/2}$ ; Energy and energy density of relativistic electrons  $E_{\text{rel}} \sim (\Delta p)c \sim \hbar n_e^{1/3} c$  and  $u_{\text{rel}} \sim \hbar c n_e^{4/3}$ .

## UNIT 13 – STELLAR REMNANTS (CONT.)

### MATHEMATICAL BASE

Pressure contributed by highly relativistic degenerate electrons  $P_{\text{rel}} \sim \frac{\hbar c}{3} n_e^{\frac{4}{3}}$ ; Central pressure of a white dwarf  $P_{\text{c,rel}} \sim \frac{\hbar c}{m_p^{\frac{3}{4}}} \frac{M^{\frac{4}{3}}}{R^4}$ ; Chandrasekhar mass for a carbon/oxygen white dwarf as  $M_{\text{ch}} \sim 1.4M_{\odot}$ ; The main sequence lifetime of a massive star  $\tau \approx 30\text{Myr} \left(\frac{M}{7M_{\odot}}\right)^{-3}$ ; Degenerate non-relativistic neutron pressure  $P_n \sim \hbar^2 \frac{n_n^{\frac{5}{3}}}{m_n}$ ; Neutron star radius  $R \sim \frac{\hbar^2}{Gm_n m_p^2} \left(\frac{M}{m_p}\right)^{-\frac{1}{3}}$ ; Ratio of neutron star to white dwarf radius/mass relation  $\sim \frac{1}{1839} \left(\frac{M_{\text{wd}}}{M_{\text{ns}}}\right)^{-\frac{1}{3}}$  yielding  $R_{\text{ns}} \sim 3 \text{ km} \left(\frac{M_{\text{ns}}}{1.4M_{\odot}}\right)^{-\frac{1}{3}}$ , and accounting for strong force  $R_{\text{ns}} \approx 11 \text{ km} \left(\frac{M_{\text{ns}}}{1.4M_{\odot}}\right)^{-\frac{1}{3}}$ ; Escape speed from a neutron star (Newtonian formula)  $v_{\text{esc}} = \left(\frac{2GM_{\text{ns}}}{R_{\text{ns}}}\right)^{\frac{1}{2}}$ ; Escape speed from a neutron star as a significant fraction of the speed of light  $\frac{v_{\text{esc}}}{c} \approx 0.6 \left(\frac{M_{\text{ns}}}{1.4M_{\odot}}\right)^{\frac{2}{3}}$ .

### ASSESSMENT

Unsupervised Individual Written (Assignment Questions – Open Book) – Units 12 & 13

### ADDITIONAL ASSESSMENT OPPORTUNITIES

- ☼ CLEA computer simulation and work-booklet “Dying Stars & the Birth of the Elements”
- ☼ CLEA computer simulation and work-booklet “Pulsars”
- ☼ GHOU “Searching & Discovering a Supernova” unit
- ☼ GHOU “Supernova Light Curves” unit

## UNIT 14 - CONIC SECTIONS

18 hours (5 weeks)

### UNIT CONTENT

Description of Conic Sections; Orbits – open (parabolic & hyperbolic), closed (circular and elliptical); Cartesian & Polar coordinate systems; Distance formula  $r = \sqrt{(x_1, y_1)^2 + (x_2, y_2)^2}$ ; Cartesian circle  $x^2 + y^2 = r^2, (x-h)^2 + (y-k)^2 = r^2$ ; Cartesian parabola  $x^2 = 4py, (x-h)^2 = 4p(y-k)$ ; Cartesian ellipse  $\frac{(x-h)^2}{a^2} + \frac{y^2}{b^2} = 1$ ; Ellipses – Properties and construction of (foci, perihelion, aphelion), perigee & apogee.

Derivation (from first principals of kinematics) of: Eccentricity as  $e = \frac{c}{a}, e = \frac{F_a - F_p}{ap}$  &  $e = \frac{A-P}{A+P}$ ; Velocities at perigee & apogee  $\hat{v}_p = \sqrt{\frac{GM}{P}(1+e)}$  &

$\hat{v}_A = \sqrt{\frac{GM}{A}(1-e)}$ ; Velocities at points in elliptical orbits  $\hat{v} = \sqrt{GM\left(\frac{2}{r} - \frac{1}{a}\right)}$ ; Period of elliptical orbits  $T = 2\pi\sqrt{\frac{a^3}{GM}}$ ; definition of eccentricity using loci,

foci and directrix; Introduction to orbital parameters - apogee and perigee (elliptical orbits); Introduction to Polar coordinates  $(r, \vartheta)$ ; Recap Sine & Cosine rules -  $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}, a^2 = b^2 + c^2 - 2bc\cos A$ ; Recap unit circle -  $x = r \cos \vartheta, y = r \sin \vartheta, \tan \vartheta = \frac{y}{x}$  functions; Converting between

Cartesian & Polar coordinate equations; Polar equations for conic sections,  $r = \frac{ep}{1 - e \cos \vartheta}$  at  $e = 1, \frac{1}{2}, \text{ \& } 2$ ;  $r = \frac{a(1 - e^2)}{1 - e \cos \vartheta}$  (ellipse);  $r = \frac{p}{2 - \cos \vartheta}$  (parabola);  $r = \frac{2p}{1 - 2 \cos \vartheta}$  (hyperbola).

### ASSESSMENT

Supervised Written (Examination) – 2 hours total

## UNIT 15 – COSMOLOGY

24hours (7 weeks)

### UNIT CONTENT

General relativity & the expanding Universe; Red-shifts & expansion; Hubble constant & the age of the Universe; Detection of the cosmic microwave background & spectrum; Symmetry of the Universe; Strong & weak forces; Quarks; Superstring theories; Evolution of the Universe – Planck time, GUT theory, inflationary epoch, confinement period; Matter & antimatter, pair production & annihilation; Primordial fireball & era of recombination; Cosmic light horizon; Structure of the early universe; Period of the dark ages & star formation – initial galaxy structures; Cosmic timeline & fate of the Universe; Dark matter & dark energy; Overall space time; Closed & open Universe; Possible geometries of the Universe; Curvature of space

fabrics; The effects of dark energy; Special theory of relativity - time dilation and length contraction  $T = \frac{T_0}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$  and  $L = L_0 \sqrt{1 - \left(\frac{v}{c}\right)^2}$ ; Space-time

continuum – Gravitational Redshift; Using general relativity to predict black holes. – Event horizons & Accretion disks; Inside a black hole ; Calculating the dimensions of black holes from the Schwarzschild radius  $R_{sch} = \frac{2GM}{C^2}$ ; Super-massive black holes – several billion  $M_{\odot}$ ; Black hole – neutron star binaries; Fate of black holes.

### ASSESSMENT

Unsupervised Written (Project) – Estimating Dark Matter in a Spiral Galaxy;

CLEA computer simulation and Exercise Questions “The Hubble Law – Redshift/Distance Relation”

CLEA computer simulation and Exercise Questions “The Large Scale Structure of the Universe”



## UNIT 16 – GALAXIES (ADDITIONAL UNIT)

14 hours (4 weeks)

### UNIT CONTENT

Hubble classification of Galaxies; Spiral galaxies; correlation between tightness of spiral arm to the nuclear bulge; Sa, Sb & Sc subclasses; The winding dilemma; Flocculent spiral galaxies; Grand design galaxies; Spiral density waves; Barred galaxies; Elliptical galaxies & subtypes; Irregular galaxies; Hubble tuning fork diagram; Clusters & Super-clusters; Globular & irregular clusters; Voids; Galactic merging & cannibalism; Galactic halos & dark matter; Red shift of galaxies; Hubble flow (expansion of universe); Hubble Law  $v = H_0 \times d$ ; Different techniques in measuring cosmological distances; Gravitational lensing; Structure of the Milky Way Galaxy; Using Cepheid's to find the Galactic centre; Using 21 cm radio radiation (photon emission from hydrogen) to map our Galaxy; Dimensions of our Galaxy; Visible versus dark matter; The Galactic nucleus; Synchrotron radiation; Orbits of halo & disk stars – presence of dark matter; Rotation of our Galaxy; Quasars (quasi-stellar objects); Large red shifts of quasars; Spectrum of quasars; Quasars as the most distant objects in the Universe; Active Galaxies; Radio galaxies; Peculiar galaxies; Seyfert galaxies; BL Lacertae objects; Blazars; Double radio sources; Head-tail sources; Galaxy & quasar luminosity table; Super-massive black holes as *central engines* that drive active galaxies; Gravity as a focus for light.

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